Application of a Digital Filter to Double Quantum Coherent Transient Phenomena*

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Abstract

The present paper describes on a simple digital filter for removing wiggles from double quantum coherent signals. A high efficiency of the digital filter is demonstrated on a double quantum transient nutation signal of ^{27}Al nuclear spins in Al_2O_3.

Basic double quantum (DQ) or two-photon coherent transient phenomena which are observed by selective excitation of a multilevel system[1] are explained by a vector model analogous to that of single quantum (SQ) coherent phenomena[2]. However, the actual DQ excitation causes another coherent effect at the same time. The coherent effect appears as a wiggle[3] in the coherent phenomena induced during the DQ excitation such as a DQ transient nutation[4]. Figure 1 shows a signal of DQ transient nutation observed in a multi-level *NMR* system of ^{27}Al nuclear spins in Al_2O_3. The oscillation signal was induced by a DQ rf field resonant to M2 transition $a \longleftrightarrow c$ with intensity of about 50 *Oe*, which was observed by measuring amounts of population difference w_{ab} in M1 transition $a \longleftrightarrow b$, where the notations a, b and c indicate the energy levels of $I_z = -3/2$, $-1/2$ and $1/2$, respectively. The experimental method for observing the signal is almost the same as that in ref. 4. A comparably large wiggle is seen in the signal. The wiggle is undesirable when we are interested only in such DQ choherent transient phenomena. It is difficult to eliminate the wiggle without any distortion of the main signal purely showing the DQ transient nutation. Filtering methods recently proposed, which are called a multiple-quantum filtering[5], have no effect on the elimination. We have designed a digital filter[6] for removing such wiggles, and have obtained satisfactory results of its application to a study of DQ rotary echo[7] due to magnetic dipole interaction[8].

In the present paper, we show in detail that the digital filter is a powerful tool for removing the wiggles from the signals of DQ coherent phenomena such as DQ transient nutations and DQ rotary echoes.

We consider two sets of values $\{x(n)\}$ and $\{y(n)\}$, representing input and output signals of the digital filter, respectively, where $n=0, 1, 2, ...$ The input signal indicates a transient signal such as the DQ transient nutation. It is assumed that the observation of the transient

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signal begins at time $t=nT=0$, and the signal is accumulated with sampling interval of T. We adopt a finite impulse response digital filter defined by

$$y(n)=\sum_{i=0}^{L-1} a_i \{x(n+L-i)+x(n-L+i)\}+a_L x(n), \tag{1}$$

where $M=2L+1$, which is odd, is the filter length, and coefficients a_i and a_L are called an impulse response. This type of filter usually produces a linear phase delay of output signal[6]. Equation (1) is modified to cancel the delay. Length M and coefficients a_i, which determine the characteristics of digital filter, are chosen to achieve the purpose by considering its frequency response, which is derived from a transfer function

$$H(z)=Y(z)/X(z), \tag{2}$$

where $X(z)$ and $Y(z)$ represent the z-transforms of $x(n)$ and $y(n)$, respectively. Letting $z=\exp\{j\omega\}$, we obtain the frequency response as

$$H(\exp\{j\omega\})=2\sum_{i=0}^{L-1} a_i cos(L-i)\omega+a_L, \tag{3}$$

where the units of time t and frequency ν are T and $1/T$, respectively, and $\omega=2\pi\nu$. Equation (3) shows that the digital filter given by Eq. (1) produces no delay. Since the n-th output value $y(n)$ is calculated from a series of the input values from $x(n-L)$ to $x(n+L)$, the initial L output values from $y(0)$ to $y(L-1)$ are not obtained from the input values, and therefore, need to be complemented by a proper extrapolation of the input signal to the area of negative time. Under the circumstances, the digital filter which is as short as possible is required.

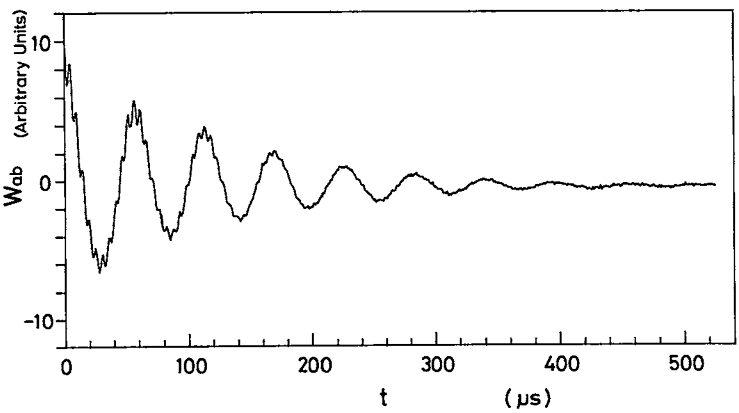

Fig. 1. Signal of DQ transient nutation induced in transition $I_z=-3/2\longleftrightarrow 1/2$ in a multi-level system of ^{27}Al nuclear spins in Al_2O_3. The oscillation was observed by measuring the amounts of population difference W_{ab} in transition $I_z=-3/2\longleftrightarrow -1/2$. The frequencies of the DQ nutation and the wiggle are about 20 *kHz* and 200 *kHz*, respectively.

We have aimed at signals as seen in Fig. 1, where the frequencies of the main oscillation and the wiggle are about 20 *kHz* and 200 *kHz*, respectively, and $T=1\mu s$. The designed digital filter is a bandstop filter with $M=13$ and

$$\begin{aligned} a_0 &= -0.043693, \\ a_2 &= 0.009904 \\ a_4 &= 0.291286, \\ a_6 &= 0.485006, \\ a_{2i+1} &= 0,\ (i=0,\ 1,\ 2). \end{aligned} \qquad (4)$$

The frequency response $H(\exp\{j\omega\})$ of the filter is shown in Fig. 2. Considering that the frequencies of the nutation signal and the wiggle are about $0.02/T$ and $0.2/T$, respectively, we can expect that the wiggle is almost completely eliminated without reduction of the nutation signal. Figure 3 shows the filtered signal w_{ab}' of the DQ nutation signal in Fig. 1. The digital filtering was performed by means of software. The complete elimination of the wiggle is recognized. Traces *a* and *b* in Fig. 4 showing the Fourier spectra of the signals in Figs. 1 and 3, visualize the fact that the elimination of wiggle is not accompanied by reduction nor distortion of the main signal. The lost initial part of output signal was complemented by adding a fictitious digital signal with six sampling intervals which is symmetric to the original one about

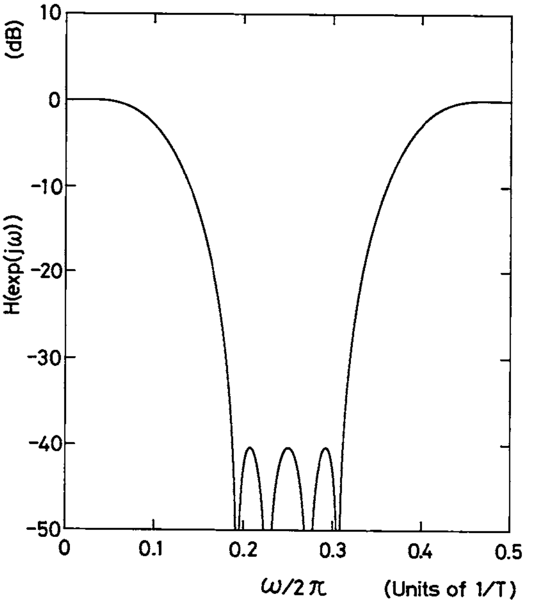

Fig. 2. Frequency response of the digital filter. The curve indicates that the digital filter reduces signals with frequency in the range of about $0.2/T$ to $0.3/T$ to less than 1% of the original intensity, but passes the ones with frequency lower than about $0.05/T$ or higher than about $0.45/T$ without reduction.

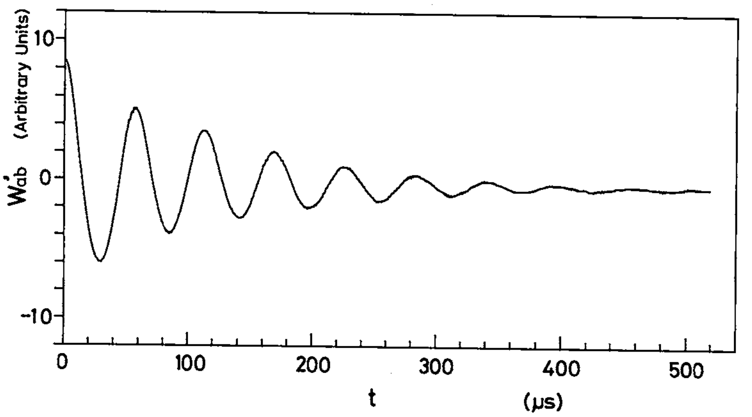

Fig. 3. Result obtained by applying the digital filter to the DQ transient nutation signal shown in Fig. 1. The wiggle is not seen at all.

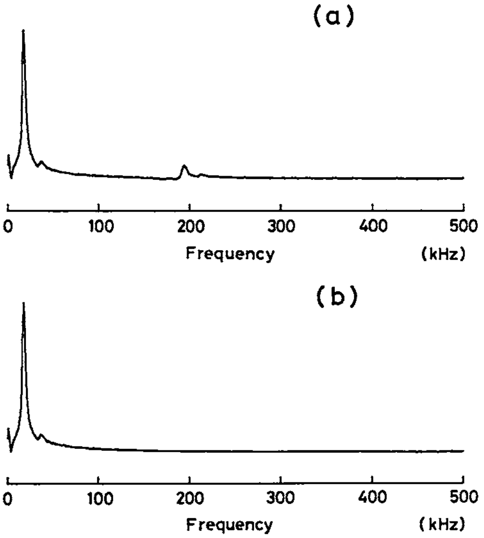

Fig. 4. Fourier spectra of the oscillation signals shown in Figs. 1 and 3. The small lines seen at about 200 *kHz* in Fig. 4a, producing the wiggle, completely disappear in Fig. 4b, while the main line at about 20 *kHz* is not distorted at all.

$t=0$ ($n=0$), that is, $x(-n)=x(n)$, $n=1,2,..6$. This procedure is valid if the decay of signal during six sampling intervals can be neglected, because the DQ nutation signal is described by periodic functions which are symmetric about $t=0$[3]. In order to examine the distortion of filtered signal in more detail, we have estimated amounts of $|y(n)-x(n)|/A$ with using a fictitious input signal indicating transient nutation without wiggle, where A is the initial amplitude of the oscillation. The amounts were less than 0.5%, which are usually within the experimental error. Therefore, the filtered signal in Fig. 3 can be regarded as the DQ nutation signal which is not distorted at all.

If the frequency of DQ rf field is shifted from the resonance, the wiggle becomes

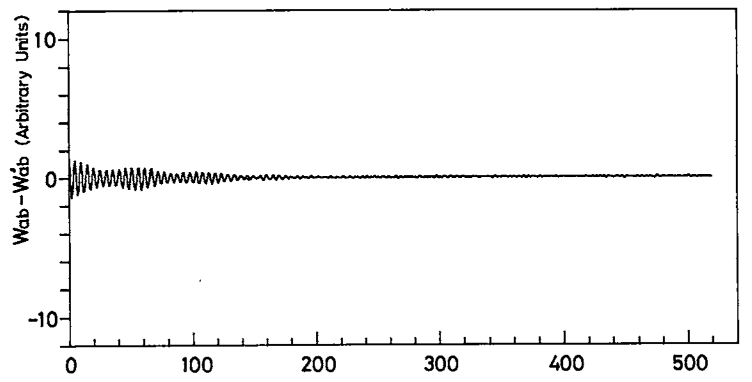

Fig. 5. Wiggle seen in the DQ nutation signal in Fig. 1. The signal was extracted by subtracting the filtered signal w_{ab}' in Fig. 3 from the original one w_{ab} in Fig. 1. It is seen that the oscillation consists of two frequencies.

larger in contrast to the main osillation showing the DQ nutation[8]. The considerably large wiggles were also completely memoved. The digital filter was also useful for observing DQ rotary echoes. With using the digital filter, it is possible to measure fairly precisely the decay times of DQ transient nutations or DQ rotary echo-envelope decays.

Figure 5 shows a result obtained by subtracting the output values indicating the signal in Fig. 3 from the corresponding input ones in Fig. 1, which shows only the wiggle. The wiggle has two frepuencies[3] as seen in Fig. 4a. The osillatory-behavior in Fig. 5 reflects the characteritsics of wiggle. The wiggle is also an interesting transient phenomenon in the three-level system. The experimental study of wiggle becomes easier with the digital filter.

The method of digital filtering is also available for other processings of transient signals, such as reduction of random noise. Wide application of this technique can be expected if the loss of the initial part of filtered signal is acceptable. An experimental study concerned in the DQ rotary echo is in progress with the digital filter.

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